TRINITY COLLEGE

Semester Two Examination, 2019

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3 AND 4

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

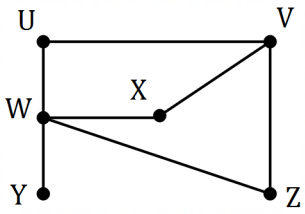
Working time: 100 minutes.

Question 9 **(6 marks)**

(a) Connected planar graph G_1 has 29 faces and 42 edges. Determine the number of vertices G_1 has. (2 marks)

(b) Draw graph G_2 so that it has 5 vertices, is a tree and is not semi-Hamiltonian. (2 marks)

(c) Graph G_3 is shown below. Redraw G_3 to clearly show that it is bipartite, underlining vertex V and all other vertices that belong to its group. (2 marks)



See next page

Question 10 **(6 marks)**

The number of tickets T_n remaining for a musical event at the start of day n can be modelled by the recursive rule $T_{n+1} = T_n - 17$. At the start of day 1, there were 646 tickets remaining.

(a) Complete the table below. (2 marks)

n	5	10	15	20
T_n				

(b) Would the graph of T_n against n appear to be linear or non-linear? (1 mark)

(c) The n^{th} term rule for this sequence is $T_n = b + an$. Determine the value of the constant a and the value of the constant b. (2 marks)

(d) At the start of day k, fewer than 50 tickets remain. State the value of k. (1 mark)

See next page

Question 11 **(11 marks)**

The table below shows the length (L) and the weight (W) of some mackerel, a species of fish.

L (cm)	52	45	39	46	36	48	41	35	43	44	49	40
W (kg)	0.94	0.78	0.45	0.77	0.32	0.79	0.59	0.37	0.64	0.61	0.88	0.57

(a) By viewing a scatterplot of this data on your calculator, identify and describe the direction and form of the relationship between the variables. (2 marks)

(b) Calculate the correlation coefficient between the variables and use it to describe the strength of their association. (2 marks)

(c) What percentage of the variation in weight can be explained by the variation in the length for this sample? (1 mark)

(d) Using length as the explanatory variable, determine the equation of the least-squares line to model the linear relationship between the variables. (2 marks)

(e) What increase in weight can be expected for each additional centimetre in length for these mackerel? (1 mark)

(f) Another mackerel in the sample had a length of 38 cm. Predict the weight of this fish and comment on two factors that support the reliability of this prediction. (3 marks)

See next page

Question 12 **(6 marks)**

A random sample of 530 TAFE students studying the same diploma was taken. Their study mode and enjoyment of the course is summarised in the table below.

		Study mode	
		Attend campus	Online
Level of course enjoyment	High	155	31
	Moderate	122	45
	Low	63	22
	None	80	12

(a) Show that the study mode of approximately 21% of the students in the sample is online. (2 marks)

(b) Complete the two-way frequency table below to show the column percentages for the above data, rounding to the nearest whole number. (2 marks)

		Study mode	
		Attend campus	Online
Level of course enjoyment	High		
	Moderate		
	Low		
	None		

(c) Explain whether the data suggests the presence of an association between the variables. (2 marks)

See next page

Question 13 **(5 marks)**

A young person has a bank account that pays no interest. At the start of each week they withdraw a fixed percentage of the balance to spend and then deposit a lump sum from their wages. The balance of the account at the end of week n is given by A_n, where

$$A_{n+1} = 0.94A_n + 33, \qquad A_1 = 65.90.$$

(a) Calculate A_4 and explain what this figure represents. (2 marks)

(b) State what percentage of the balance they withdraw each week. (1 mark)

(c) The young person plans to continue operating the account in this manner until it holds at least $\$600$. Comment on this plan. (2 marks)

See next page

Question 14 **(6 marks)**

An investor is considering placing the sum of $\$96\,000$ into one of three investments for 2 years.

Investment A offers a return of 6.82% per annum compounded daily.

(a) Determine the profit that the investor would achieve through investment A. (2 marks)

Investment B offers a return of 6.83% per annum compounded monthly.

(b) Calculate the difference in profit that the investor would achieve through investment B compared to investment A. (2 marks)

Investment C offers a return of 6.87% per annum compounded quarterly.

(c) Rank the three investments in order, from highest to lowest profit. Justify your answer. (2 marks)

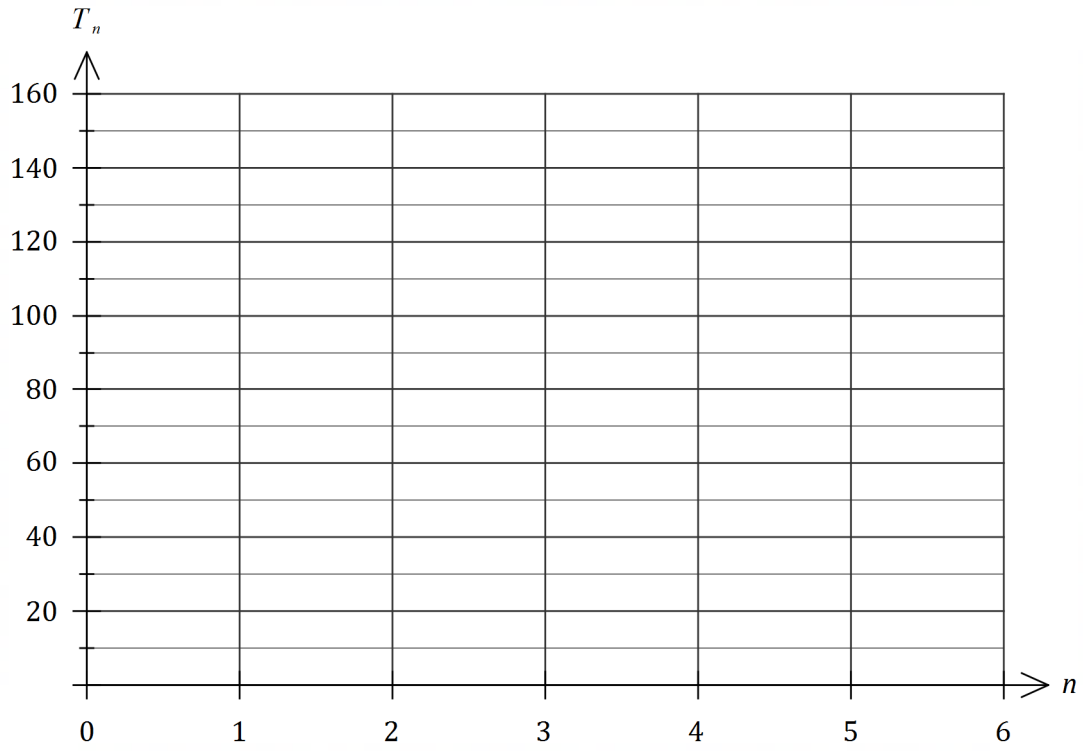
See next page

Question 15 **(6 marks)**

An electric kiln is switched on and after n minutes, the temperature of the kiln T_n is given by the recursive rule $T_{n+1} = 1.35T_n$. The initial temperature of the kiln, T_0, is $25°C$.

(a) Calculate the temperature of the kiln after 1 minute. (1 mark)

(b) Graph the temperature of the kiln for $n = 0, 1, 2, 3, 4, 5$ and 6 on the axes below. (3 marks)



(c) Name the type of growth displayed in the graph. (1 mark)

(d) During which minute does the temperature of the kiln first increase by more than $200°C$? (1 mark)

See next page

Question 16 **(8 marks)**

A delivery driver must leave depot D at $8{:}30$ am, deliver packages to three schools (A, B and C) and then return to the depot. The table below shows the travel times between the various locations in minutes.

	A	B	C
D	21	22	24
C	23	26	
B	25		

(a) Construct a weighted graph to represent this information. (3 marks)

(b) Explain why the graph in part (a) is a complete graph. (2 marks)

(c) Determine the route the driver should take to minimise delivery time and calculate the time they will arrive back at the depot if they spend 7 minutes at each school handing over the packages. (3 marks)

See next page

Question 17 **(8 marks)**

A customer in a store is offered a reducible interest loan that attracts interest of 7.2% per annum compounded monthly to purchase a $\$3\,209.75$ computer and accessories. The monthly loan repayment is $\$144$.

The customer set up the spreadsheet below to analyse the loan, rounding the displayed figures to the nearest cent.

Month	Balance at start of month ($)	Interest for month	Repayment	Balance at end of month ($)
1	3 209.75	19.26	144.00	3 085.01
2	3 085.01	18.51	144.00	2 959.52
3			144.00	

(a) Complete row 3 of the spreadsheet. (3 marks)

(b) Determine a simplified recurrence relation for B_n, the loan balance at the **start** of month n. (2 marks)

(c) State the number of repayments required to pay off the loan. (1 mark)

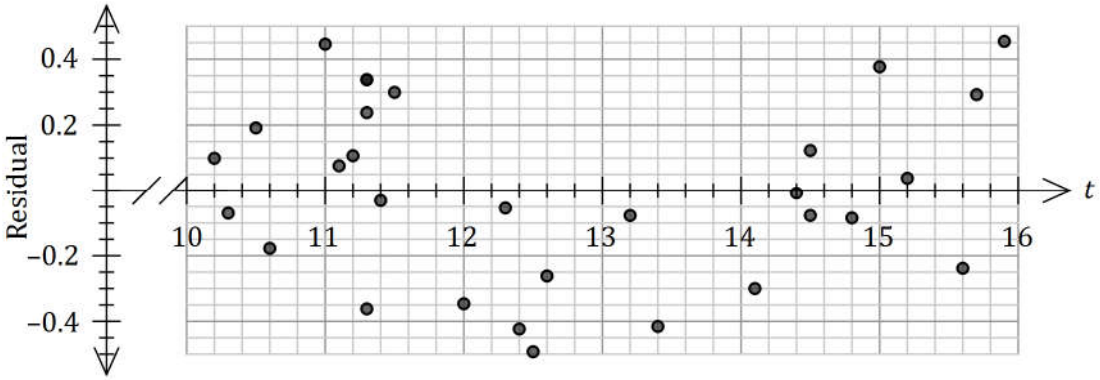
(d) Determine the total amount of interest paid over the life of the loan. (2 marks)

See next page

Question 18 **(9 marks)**

An experiment involved measuring the voltage v in a circuit after t seconds. The equation of the least-squares line to model the linear relationship between v and t is given by $v = 5.54 - 0.31t$ and $r_{vt} = -0.935$.

The residual plot for the linear model is shown below.



(a) After 10.4 seconds, the measured voltage was 2.72. Calculate the residual for this point and add it to the plot above. (3 marks)

(b) Explain what information in this question

(i) supports the use of the linear model. (2 marks)

(ii) does not support the use of the linear model. (2 marks)

(c) The residual for the voltage measured after 12 seconds is shown on the plot above. Determine what voltage was measured at this time. (2 marks)

See next page

Question 19 **(7 marks)**

An annuity compounds interest annually and its value after n withdrawals can be modelled using the recurrence relation

$$T_{n+1} = 1.054T_n - 37\,500, \qquad T_0 = 478\,000.$$

(a) Use the relation to state

(i) the annual percentage interest rate. (1 mark)

(ii) the initial value of the annuity. (1 mark)

(b) Calculate the balance of the annuity, to the nearest dollar, after 4 withdrawals. (1 mark)

(c) The annuity is closed after 15 withdrawals. Calculate the total interest paid by the annuity up to this time. (2 marks)

(d) From the outset, the annual withdrawal can be reduced so that the annuity becomes a perpetuity. Briefly explain what a perpetuity is and determine the withdrawal required. (2 marks)

See next page

Question 20 **(10 marks)**

A reverse mortgage is a loan that allows a person to borrow money using their home as security. Interest is charged like any other loan, but no repayments are made - the interest compounds over time and is added to the loan balance.

At the start of January 2019, a borrower takes out a reverse mortgage for $\$173\,000$ at an interest rate of 7.44% per annum, compounded monthly.

(a) State a simplified recurrence relation for T_n, the loan balance after n months. (2 marks)

(b) Determine the loan balance at the start of January 2023. (2 marks)

(c) Determine the effective interest rate of the reverse mortgage, correct to 3 decimal places. (1 mark)

See next page

The borrower's home was valued at $\$520\,000$ at the time they took out the reverse mortgage and was expected to appreciate at a rate of 2.8% per annum.

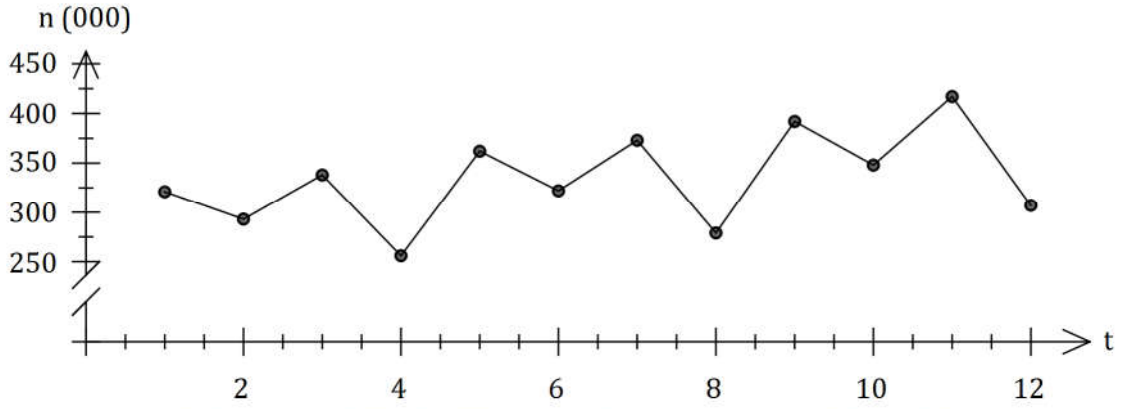
(d) Use a recurrence relation to determine the value of the home at the start of January 2023. (2 marks)

(e) At the start of which calendar year will the loan balance first exceed the value of the home? Justify your answer. (3 marks)

See next page

Question 21 **(10 marks)**

The number of people (n, in thousands) studying in Australia with a student visa from 2014 to 2016 are shown in the graph and table below.



Year	Quarter	Time (t)	Number (n 000's)	Quarterly mean	Percentage of quarterly mean
2014	1	1	321	A	106.3
	2	2	293		97.0
	3	3	338		111.9
	4	4	256		84.8
2015	1	5	B	334	108.4
	2	6	322		96.4
	3	7	373		111.7
	4	8	279		83.5
2016	1	9	392	366	C
	2	10	348		95.1
	3	11	417		113.9
	4	12	307		83.9

(a) Determine the value of A, the value of B and the value of C in the table above. (3 marks)

See next page

(b) Complete the missing values in the seasonal index table below. (2 marks)

Quarter	1	2	3	4
Seasonal Index	1.072	0.962		

(c) Calculate the deseasonalised figure for the number of people studying in Australia with a student visa in the second quarter of 2016. (2 marks)

Let N represent all the deseasonalised values of n. The equation of the least-squares line for N against t is given by $N = 7.11t + 287.8$.

(d) Determine an estimate for the number of people studying in Australia with a student visa in the second quarter of 2018 if the existing trend and seasonality continues. (3 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____